Constructing a Parabola: Lesson Plan

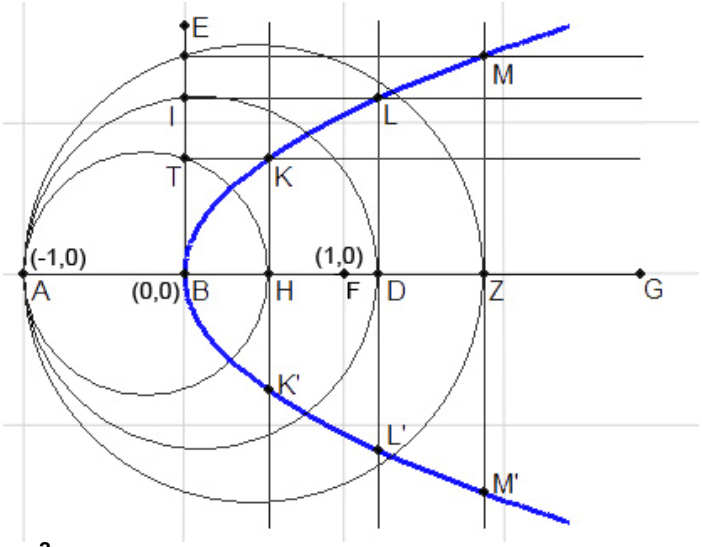
Overview:
Students will learn about the rich history of Islamic Mathematics, and will gain an understanding of the relationship of Islamic mathematics to modern mathematics. This unit requires some basic knowledge of geometry, and so is geared towards high school students or middle-school students learning about parabolas and geometrical figures. Students will learn one of the methods used by Muslim mathematicians in constructing parabolas. Using the history intertwined with the mathematics lesson, students will be more interested in the lesson and have a better understanding of parabolas.

Objectives:
Students will be able to:
1. Recount a brief history of Islamic mathematics and the expansion of Greek mathematics in the Arab world
2. Construct a parabola
3. *Optional*: Construct a geometric proof

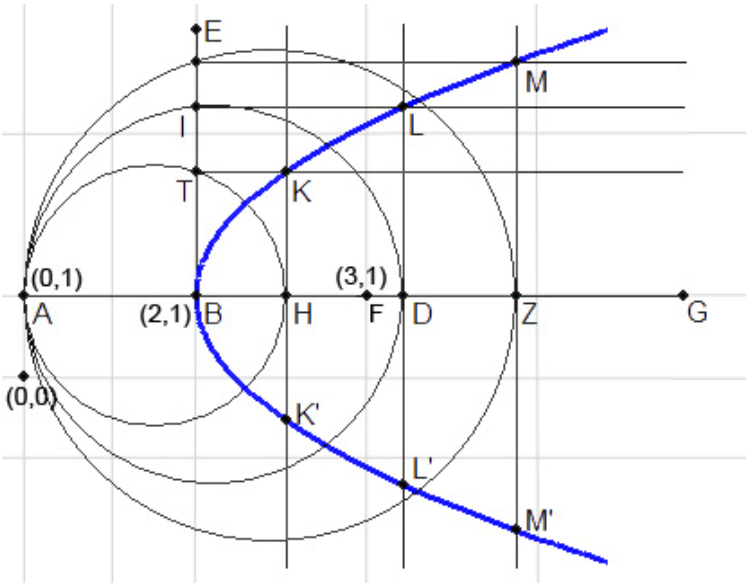
Activity:
Opening / Hook:
1. Students should be familiar with geometric figures, such as curves and hyperbola.
   Definitions:
   - A **parabola** is the set of all points in the plane equidistant from a given line L (the conic section directrix) and a given point F not on the line (the focus). *Definition from Wolfram MathWorld*
   - A parabola has one **focus** and a **directrix** (line) such that the distance from any point of the parabola to the focus is equal to the (perpendicular) distance from the point to the directrix. *from Wikipedia, focus*
   - The **axis** of the parabola passes through the focus and is perpendicular to the directrix.

Introduce New Material:
1. First, show parabolas drawn on the board, showing its vertex, axis, and parameter, which is the distance from vertex to the focus, or vertex to the directrix (by definition of the focus, this distance is equal). Also provide its equation, with vertex (x, h) and parameter p, in the forms:
   $(x-h)^2 = 4p(y-k)$ – where the parabola's axis is parallel to the x-axis
   $(y-k)^2 = 4p(x-h)$ – where the parabola's axis is parallel to the y-axis
   You may wish to use the following examples:

$x^2 = 4y$, with focus F, vertex B, parameter AB (=1)



$(x-2)^2 = 8(y-1)$, with focus F, vertex B, parameter AB (=2)



2. Give a brief history of Islamic mathematics (pp. 1-2 in "Islamic Mathematics" information packet). Also, describe the contributions of Muslim geometers like Ibn Sinān to geometry. (pg 6, "Islamic Mathematics" information packet)
3. Show the construction of a parabola using Ibn Sinān's method (pp 6-8, "Islamic Mathematics" information packet). It would be helpful for students if you showed the construction of one or both of the two example parabolas.
4. *Optional*: For high-school geometry students (learning proofs): prove (or ask them to prove) that the points lie on the parabola; i.e. that the construction is a parabola. (pp 8 "Islamic Mathematics" information packet)

Guided Practice:

1. Pass out the Parabola Construction worksheet, and help students to complete the first problem by completing it on the board.

Independent Practice:

1. Then, have students work in groups of 2-3 people to complete the rest of the worksheet.

Closing / Assessment:

1. For homework, assign problems similar to these (from a textbook or worksheet), or perhaps assign a few equations, asking students to identify the equations as parabolas, circle, hyperbolas, or ellipses (whichever figures have already been taught), and students to draw the corresponding figures (using Ibn Sinān's construction of a parabola at least once on the homework). You may wish to also provide a few parabolas and ask students to identify the corresponding equations.

Teachers: Please contact Angela Williams, CSAMES Outreach Coordinator, at aswillms@illinois.edu for the answer key to the worksheet.